Seat No. _______________

HQ-8

B. Sc. (Sem. II) (CBCS) (W.E.F. 2019) Examination

April - 2023

Mathematics : BSMT-02(A)

(Geometry, Calculus and Matrix Algebra)

(New Course) (Theory)

Time : $\mathbf{2\frac{1}{2}}$ / Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures written to the right side indicate full marks of the question.

1 (a) Answer the following questions briefly : **4**

(1) Define Vector form of Sphere.

(2) Find the Sphere for which (1, 1, 0) and (0, 1, 1) are the extremities of a diameter.

(3) Define Right Circular Cylinder.

(4) Write the equation of cylinder with generator is parallel to Y - axis and enveloping curve is $x^2 + y^2 + z^2 = a^2$.

(b) Answer any one briefly : **2**

(1) Find centre and radius of sphere

$x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$.

(2) Find equation of right circular cylinder with radius 2 and axis $\dfrac{x-1}{2} = y - 2 = \dfrac{z-3}{2}$.

(c) Answer any one briefly : **3**

(1) Derive the equation of tangent plane and normal to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ which is parallel to plane $3x + 2y - 2z = 0$.

(2) Find equation of cylinder whose generator is parallel to $x = y = z$ and enveloping the curve is

$x^2 + y^2 + z^2 = a^2$.

(d) Answer any one briefly : **5**

(1) Derive the equation of tangent plane and normal to the sphere $x^2+y^2+z^2=a^2$ at the point (p, q, r).

(2) Derive the equation of cylinder of which generator remain parallel to the line $x=y=z$ and passing through a guiding curve

$$ax^2+2hxy+by^2+2gx+2fy+c=0,\ z=0.$$

2 (a) Answer the following questions briefly : **4**

(1) Define Iterated Limit.

(2) Define Continuity of two variable function.

(3) Define Partial Differentiation.

(4) State Young's Theorem.

(b) Answer any one briefly : **2**

(1) Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$

(2) If $x^3+y^3+z^3=3xyz$ then find $\dfrac{\partial z}{\partial y}$.

(c) Answer any one briefly : **3**

(1) Prove that $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x+y)}{x+y}=1$.

(2) If $z=f(x+ky)+g(x-ky)$ then prove that

$$\frac{\partial^2 z}{\partial y^2}=C^2\frac{\partial^2 z}{\partial x^2}.$$

(d) Answer any one briefly : **5**

(1) Find f_x and f_y for $f(x,y)=\begin{cases}\dfrac{x(x^2-y^2)}{x^2+y^2} & \text{if } (x, y)\neq(0, 0)\\ 0 & \text{if } (x, y)=(0, 0)\end{cases}$

using the definition of partial derivatives.

(2) If $f(x,y)=0$ then obtain $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

3 (a) Answer the following questions briefly : **4**

(1) Define local minima.

(2) Define global minima.

(3) Define Extreme point.

(4) Define Jacobian.

(b) Answer any one briefly : **2**

(1) If $x = \cos\theta$ and $y = \sin\theta$ then prove that $\frac{\partial(x, y)}{\partial(r, \theta)} = 1.$

(2) State Maclaurian's expansion for function of several variables.

(c) Answer any one briefly : **3**

(1) If $f(x, y) = x^3 + xy + y^3$ then find approximate value of $f(1.01, 2.98)$.

(2) If $x = 2\cos\theta,\ y = 2\sin\theta$ and $z = z$ then prove that Jacobian $J\left(\frac{x,\ y,\ z}{r,\ \theta,\ z}\right) = 2.$

(d) Answer any one briefly : **5**

(1) State and Prove Taylor's expansion for function of several variables.

(2) Find the value of the greatest rectangular parallelepiped that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$

4 (a) Answer the following questions briefly : **4**

(1) Define Involuntary Matrix.

(2) Define Idempotent.

(3) Define Linearly Independent Set of Vectors.

(4) Define Rank of Matrix.

(b) Answer any one briefly : **2**

(1) Prove that $M = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ is nilpotent of index 2.

(2) Define Hermitian and Skew – Hermitian Matrix.

(c) Answer any one briefly : **3**

(1) Prove that matrix $M = \begin{bmatrix} \cos x & 0 & \sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix}$ is orthogonal.

(2) Using Cayley – Hamilton theorem find inverse of matrix

where $M = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

(d) Answer any one briefly : **5**

(1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew – symmetric matrix.

(2) Let $A = [a_{ij}]_{1 \le i,\ j \le n}$ and $B = [b_{ij}]_{1 \le i,\ j \le n}$ then prove that $abj(AB) - adj(b)\ adj(A)$.

5 (a) Answer the following questions briefly : **4**

(1) Define Characteristic Polynomial.

(2) Define Eigen Vectors.

(3) What is the determinant of odd order skew – symmetric matrix?

(4) Define Non – Homogeneous system of linear equation.

(b) Answer any one briefly : **2**

(1) Verify Cayley – Hamilton theorem for matrix

$M = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(2) Prove that Eigen values of Symmetric matrix is real numbers.

(c) Answer any one briefly : **3**

(1) Find Eigen values and Eigen vectors of matrix

$M = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$.

(2) Solve $x + y + 2z = 9$, $2x + 4y - 3z = 1$ and $3x + 6y - 5z = 0$.

(d) Answer any one briefly : **5**

(1) State and Prove Cayley – Hamilton Theorem.

(2) The necessary and sufficient condition that the system of equation $AX = B$ is consistent is that the matrices A and $[A : B]$ are of the same rank.